Abstract

Constructivism is one of the most important ideas in current educational psychology. If I had to reduce all of educational psychology in learning to just one principle I would say this: the most important single factor influencing learning is what the learner already knows. According to constructivism, knowledge cannot be transmitted and teachers cannot simply give students knowledge. Instead, students' knowledge must be constructed in their own minds. Today, many constructivist mathematics educators would maintain as a central tenet that the mathematics children know should be the basis on which to teach mathematics. Our existing schemas serve either to promote or restrict the association of new concepts and affecting their ability to understand. This process of mental construction involves two director systems, first is delta-one (Δ₁) a kind of sensory-motor system which “receives information and second one is delta-two (Δ₂) is a goal directed mental activity (Skemp, 1979a). In assessing students in mathematics understanding, a problem we face is that we are all too often assessing only a limited part of their understanding. For example, when asking a student to carry out a multiplication calculation, are we really assessing their understanding of multiplication? To be clear about how we do this, we need to be clear about understanding itself. The contribution of this paper is to clarify this link between assessment and understanding. The aim of this theoretical paper is to examine the different ways in which we can examine students’ understanding in mathematics. In order to do so, we begin with constructivism in mathematics, defining exactly what we mean by understanding in this context, before moving on to examine what this means for the methods of assessment that we can employ. This will have implications for how we carry out research into students’ understanding of mathematical concepts. Therefore, this paper begins by providing an overview of what we mean by this concept of constructivism and how can we assess mathematical understanding.

Keywards: Constructivism, assessment, mathematical understanding.

INTRODUCTION

Constructivism theory is not a new, it has a long history in philosophy with some aspects dating from Xenophanes, a 6th century B.C. sceptic, who said that if someone described exactly how the world really was, they would have no way of knowing that it was a true description (Von Glasersfeld, 1990). This major argument of the sceptics for the last 2500 years assumes that all ideas and knowledge gained is derived from our experiences - our senses, our acting and our thinking. Such ideas continued to emerge in debates between objectivists who believe that things can only be known objectively and that knowledge is outside the mind, and the relativists who
believe that all knowledge is subjective. These ideas were made explicit by Vico who lived from 1688 to 1744 and wrote on the construction of knowledge (Von Glasersfeld, 1990), he said “the human mind can only know what the human mind has made”. While the sceptics, Vico, and other thinkers are sources of constructivism, Piaget is “the great pioneer” (Von Glasersfeld, 1990). In summarising Piaget’s contributions Von Glasersfeld argues for two principles:
A. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.
B. The function of cognition is adaptive, in the biological sense of the term, tending toward fit or viability.
C. Cognition serves the subject’s organisation of the experiential world, not the discovery of an objective reality.

Apart from Piaget (1937), the other pioneer of constructivism is Vygotsky (1962). His ideas are generally called constructionism, or social or socio-cultural constructivism. Vygotsky believed in the primacy of culture in shaping development. His view of instruction was that interaction with adults or more advanced peers was necessary for development and that this required the active involvement of all participants. He assumed that interaction influenced development, that the teacher can intentionally nurture and teach children only in collaboration with them, and that this process requires the teacher to move ahead of development into what he called a “zone of proximal development”.

In reality, no one can teach mathematics. Effective mathematics teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding and constructivist theories are about ‘knowledge and how we come to know’. On one hand, constructivist instruction gives preeminent value to the development of students’ personal mathematical ideas. On the other hand, traditional instruction values only established mathematical techniques and concepts. In contrast, in constructivist instruction, students are encouraged to use their own methods for solving problems. They are not asked to adopt someone’s thinking but encouraged to refine their own through interaction with mathematical tasks and other students, the student’s own intuitive mathematical thinking gradually becomes more abstract and powerful.

The constructivist teacher, by offering appropriate tasks and opportunities for dialogue, guides the focus of students’ attention, thus unobtrusively directing their learning (Bruner 1986).

### Constructivism and goals of constructivism

Most traditional mathematics instruction and curricula are based on the transmission, or absorption, in view of teaching and learning. In this view, students passively “absorb” mathematical structures which invented by others and recorded in texts or known by authoritative adults. Teaching consists of transmitting sets of established facts, skills, and concepts to students. Constructivism offers a sharp contrast to this view. This process of mental construction involves two director systems, which Skemp (Skemp, 1979a) describes as delta-one ($\delta_1$) and delta-two ($\delta_2$). The former is a kind of sensori-motor system which “receives information... compares this with a goal state, and with the help of a plan which it constructs from available schemas, takes the operand from its present state to its goal state.” (Skemp, 1979b). On the other hand $\delta_2$ is a goal directed mental activity, whose operands are in delta-one, and its job is to optimise the functioning of delta-one (Skemp, 1979a). Hence the construction of concepts in a schema, or knowledge structure, may be by abstraction via direct sensory experience from actuality (primary concepts) using $\delta_1$, or by derivation from other concepts (secondary concepts), using $\delta_2$. In turn, the acquisition of new concepts may require expansion or re- construction of the relevant schema, altering it to take account of a concept for which it is relevant but not adequate (Skemp, 1979c.). Skemp (1979b) outlines two modes of mental or ‘intelligent’ activity which take place in the context of $\delta_1$ and $\delta_2$: namely intuitive and reflective:

In the intuitive mode of mental activity, consciousness is centred in $\delta_1$. In the reflective mode, consciousness is centred in $\delta_2$. ‘Intuitive’ thus refers to spontaneous processes, those within $\delta_1$, in which $\delta_2$ takes part either not at all, or not consciously. ‘Reflective’ refers to conscious activity by $\delta_2$ on $\delta_1$.

So by Skemp’s director system and the mental activities of the students, we can made some basic tenets regarding knowledge construction.

I. Knowledge is actively created or invented by the child, not passively received from the environment. This idea can be illustrated by the Piagetian position that mathematical ideas are made by children, not found like a pebble or accepted from others like a gift (Steffe and Cobb 1988).

II. Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are constructed or made meaningful when children integrate them into their existing structures of knowledge.

III. Learning is a social process in which children grow into the intellectual life of those around them (Burner...
IV. When a teacher demands that students use set of mathematical methods, the sense-making activity of students is seriously curtailed. Students tend to mimic the methods by rote so that they can appear to achieve the teacher’s goals. Their beliefs about the nature of mathematics change from viewing mathematics as sense making to viewing it as learning set procedures that make little sense.

Goals of constructivism

Although it has many different interpretations, where constructivism is concern there are some goals for mathematics instruction (Cobb 1988). Here I intend to fix some major goals in mathematics teaching through constructivist approach. First, students should develop their mathematical structures that are more complex, abstract, and powerful than the ones they currently possess so that gradually they will be able to solve a wide variety of meaningful problems.

Second, students should become autonomous and self-motivated in their mathematical activity. Students’ should believe that mathematics is a way of thinking and thinking is the only way to solve mathematical problems. Their believe will be change so that they do not "get" mathematical knowledge from their teacher so much as from their own explorations, thinking, and participation in discussions.

Also another important goal of the constructivist approach in mathematics is assessment, measurement and evaluation what should be a natural part of the learning process rather than an activity completed at the end of the learning process. Additionally, students should be involved in making judgments of learning and these judgments should be combined with judgments of teachers or other experts or peers regarding their constructing knowledge or understanding of mathematics. So the goals of constructivism approach are to hike the confidence level of the students.

Implications for learning

The theories that learners’ knowledge construct are provisional. The idea that any theory, schema or model that one constructs about any topic is provisional follows from the idea that learning is personally constructed and that cognition (coming to know) is an adaptive process. Thus, if one constructs an idea to make sense of one’s world, then a new experience may result in a need to change one’s theory.

In view of learning through constructivism we can say, knowledge construction is an iterative process through learning. Today’s knowledge is the bases of tomorrow’s knowledge. So when constructivism is concern, we can say there are five steps for constructing new knowledge. That are-

I. Every learner has ideas prior to learning and these affect the way that they make sense of what they are going to learn (previous knowledge),

II. Learning is not transmitted by linguistic communication but language is a tool to help students construct knowledge (communicating language),

III. Learning is a participatory process (active participation).

IV. Individual constructions should fit with the accepted views of communities of practice (accepted views),

V. Knowledge is personally constructed from new experiences (knowledge construction),

Teaching approaches and role of Teacher

The constructivist theory is based on the principle of encouraging students to confront, construct and develop new knowledge by actively taking part in the teaching and learning process through social interactions. For students to develop and create their own knowledge, they have to participate in a series of activities designed by a knowledgeable adult which aim to provide the individual student with varying opportunities to learn. According to Zhao (2003), the “characteristics of constructivist teaching models include: prompting students to observe and formulate their own questions; allowing multiple interpretations and expressions of learning; encouraging students to work in groups; and in the use of their peers as resources to learning”.

So, constructivist teaching methods are intended to induce learning through discovery and investigation, classified as open or discovery methods. Constructivist teaching methods are student-centred, with an emphasis on the creation of an enabling environment for the student to explore and develop new knowledge. Group work methods, discovery or investigative method and a learner-focused approach are the strategies distinguishable in the literature.

Here I discussed some of teaching approaches which are very effective and useful in constructivist approach.

In constructivist teaching, interactive teaching approach is very much useful to construct knowledge of the students with their own pace. In this approach teacher endeavours:

- To become more sensitive to learner’s ideas and questions and provide exploratory experiences from which the learners will raise useful questions and suggest sensible explanations;
- To carry out with the whole class or with groups of learners, activities to focus on the questions and ideas that many of the learners had;
- To act as a team research leader with the class, to help them plan and carry out their own investigations into their questions, and to help them draw sensible and useful conclusions from their findings. This requires the teacher to develop the skill of interacting with the learners to challenge, modify and extend their ideas, instead of providing 'right' answers and leaving the students to make sense of their experiences.

Also problem-centred teaching method is another useful way of constructivist teaching approach. Here- the teacher selects tasks which have a high probability of being problematical for students;
- The learners work on these tasks in small groups or individual. During this time the teacher facilitates collaborative work as a goal as the social interaction that occurs is seen as beneficial;
- The class is reconvened as a whole for sharing. They present their solutions to the class, not to the teacher, for discussion. The role of the teacher in these discussions is that of facilitator and every effort is made to be non-judgemental and encouraging. This stance is used so that students are empowered and in control of their learning.

Moreover a numerous teaching approaches can use in constructivist ideas like group work approach, discovery or investigative approach, individual teaching approach etc. Whatever be the constructivist approach be by naming the main five components are needed exclusively or inclusively in all the approaches. Those are:
- Orientation (to arouse interest and curiosity);
- Elicitation of ideas (to help children clarify what children think);
- Restructuring of ideas (to encourage children to clarify and share ideas, to expose them to cognitive conflict situations, to facilitate construction of and to evaluate new ideas);
- Application of ideas (to help children relate what they have learned to their everyday lives);
- Review change in ideas (to compare with previous ideas).

Role of the teacher

In these and other 'constructivist' approaches there is a role shift for the teacher which moves them from 'sage on the stage' to 'guide on the side'. This shift is likely to involve:
I. Teacher serve in the role of friends, mentors, coaches and facilitators;
II. Negotiating the details of what is to be taught;
III. Valuing the learners' ideas and their autonomy;
IV. Finding appropriate challenging problems and learning activities;
V. Emphasising cooperation in learning;
VI. Encouraging communication as a form of social interaction;
VII. Trying to find what is going on inside the heads of learners rather than relying on their overt and often superficial responses;
VIII. Taking an interest in the errors (alternative conceptions) which may throw light on how the learner is deviating from the teacher's intended path;
IX. Helping the students make connections by linking what is being taught with prior knowledge and experiences, with other parts of the subject, with other subjects, and with life outside school;
X. Accepting the notion that learning is skill developmental, and that children's learning may differ with age as their thinking is constrained because certain higher intellectual functions including awareness of mental operations are not available until adolescence;

Assessment of mathematical understanding

In mathematics education we can say “One knows more about mathematics implies one understand better the mathematics”. We begin with the definition or explanation 'understanding' in mathematics. Skemp (1976) identified two types of understanding; relational and instrumental. He described relational understanding as “knowing both what to do and why”, and the process of learning relational mathematics as “building up a conceptual structure”. Instrumental understanding, on the other hand, was simply described as "rules without reasons". The fact that most of the cases in our society students' mathematical understanding are instrumental type.

As we know, mathematics is a hand work activity. So, we can say assessing of mathematical understanding is an action or is a result of an activity. This activity is the reflection of mental experience which is associated with linking what is to be understood with the 'bases' for the understanding. The bases are mental representations, mental models, and memories of past experiences. So, 'understanding' which is acquired as a result of the acts of understanding and the 'processes of understanding' which involve links being made between acts of understanding through reasoning processes, including developing explanations, learning by example, linking to previous knowledge, linking to figures of speech and carrying out practical and intellectual activities. So the assessment is a continuous form. And the focus of the assessment would be to:
- Find what learners bring to the class in terms of their interests, prior knowledge and ideas, alternative conceptions and likely questions;
- Get an indication of how their new knowledge and their prior knowledge are interacting;
- Find what the learners are thinking rather than their overt responses, that is their constructed meanings
rather than whether they have the 'right' answer;
- Measure understanding rather than training, for example measure the ability to use simple ideas in unfamiliar situations rather than the ability to recall facts or use procedures in set and familiar situations.

More over students’ mathematical understanding occurs at three different levels i.e. concrete, representational and abstract level. Most of assessment in mathematics occurs solely at the abstract level of understanding. Concrete level of understanding is the lower level of understanding; during assessment in this level students’ demonstrate their understanding using objects. In representational level students shows their understanding through drawing and using picture. And the highest level of mathematical understanding is the abstract level of understanding; here students able to demonstrate an abstract level of understanding e.g. using only numbers and mathematical symbols, problem solve in their head. So when students can able to show his abstract level of understanding obviously they are master in other two levels of understanding.

CONCLUSION

In this paper, I have used the existing literature to obtain a view of constructivism in mathematics teaching. Constructivist theories are about 'how one comes to know'. Today’s constructing knowledge is tomorrows prior knowledge to construct another knowledge i.e. learners constructing knowledge are provisional. There are five basic tenets (previous knowledge, communicating language, active participation, accepted views and knowledge construction) in implication in constructivist learning. Constructivist teaching approach is the challenging one to teaching mathematics. No particular constructivist teaching approach is available to teach mathematics, here I have discussed some methods like interactive teaching approach, problem centred teaching approach may be the best approach in constructivist theory and the role of teacher is some different than other theory. Finally using the existing literature of mathematical understanding I have tried to correlate formative evaluation with the mathematical understanding. I would hope that we all encourage existing and prospective teachers to consider constructivism while at the same time we remain vigilant in looking for better theories.

REFERENCES